

New solutions of exotic charged black holes and their stability

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Abstract

We find a class of charged black hole solutions in third order Lovelock Gravity. To obtain this class of solutions, we are not confined to the usual assumption of maximal symmetry on the horizon and will consider the solution whose boundary is Einstein space with supplementary conditions on its Weyl tensor. The Weyl tensor of such exotic horizons exposes two charge-like parameter to the solution. These parameters in addition with the electric charge, cause different features in compare with the charged solution with constant-curvature horizon. For this class of asymptotically (A)dS solutions, the electric charge dominates the behavior of the metric as r goes to zero, and thus the central singularity is always timelike. We also compute the thermodynamic quantities for these solutions and will show that the first law of thermodynamics is satisfied. We also show that the extreme black holes with nonconstant-curvature horizons whose Ricci scalar are zero or a positive constant could exist depending on the value of the electric charge and charged-like parameters. Finally, we investigate the stability of the black holes by analyzing the behavior of free energy and heat capacity specially in the limits of small and large horizon radius. We will show that in contrast with charged solution with constant-curvature horizon, a phase transition occurs between very small and small black holes from a stable phase to an unstable one, while the large black holes show stability to both perturbative and non-perturbative fluctuations.

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I. INTRODUCTION

One of the predictions of the braneworld with large extra-dimensions is the possibility of higher-dimensional black hole. A reason for constructing higher-dimensional theories of gravity is that they provide a framework for unifying gravity with other interactions. String theory as one important candidate for such a unified theory, also predicts higher-curvature corrections to general relativity in addition to the existence of extra dimensions [1]. For decades we know that Einstein-Hilbert action is only an effective gravitational action valid for small curvature or low energies and should be modified by higher-curvature terms. Lovelock theory of gravity [2] is the most appropriate one in which the equation of motion continues to remain second order avoiding ghosts. Lovelock theory suggests that in higher than four dimensions, higher order terms have to be added to the usual Einstein-Hilbert action in order to preserve the unique properties of general relativity in four dimensions. These higher order gravity terms, are dimensionally extended Euler Poincaré densities of two, four-dimensional and so forth manifolds. Most of the researches on this subject has been concerned with second-order Lovelock gravity known as Einstein-Gauss-Bonnet (EGB) gravity, in which terms quadratic in the curvature are added to the action [3]. In third-order Lovelock gravity, Lagrangian and field equations look complicated, but particular features arising in the solutions attracts the interests to deal with the higher curvature terms in this theory. There also exist a large number of works on introducing and discussing various exact black hole solutions of third order Lovelock gravity [4–8]. Recently, some works have been extended to general Lovelock gravity to investigate the solutions and their properties [9–12]. Most of the researches have been done to derive solutions with maximally symmetric horizons. A generalization comes through the consideration of horizons which belong to the more general class of Einstein spaces. In four dimensions, the first explicit inhomogeneous compact Einstein metric was constructed by Page [13] and then was generalized to higher dimensions [14]. After that Bohm constructed metrics with non-constant curvature on products of spheres [15] and examples in higher-dimensional spacetimes has been worked on [16–21]. One of the advantages of considering higher curvature terms in Lovelock gravity is obtaining new static solution with nonconstant curvature horizon. In general relativity, substituting the usual $(n - 2)$ -sphere of the horizon geometry for an n -dimensional space-time, with an $(n - 2)$ -dimensional Einstein manifold will not alter the black hole potential

because Einstein's equations only involve the Ricci tensor. The presence of the Lovelock terms expose the Riemann curvature tensor to the equations and the new solution seems to be obtained due to the appearance of the Wyle tensor in the relation for the Riemann tensor of an Einstein space. In [22], the authors considered a static spacetime with generic Einstein space as dimensional subspace and found that only horizons satisfying the appropriate conditions on $C_{acde}C_{bcde}$ are allowed, where C_{abcd} is the Weyl tensor. This constraint appears in the metric and changes the properties of the spacetime. Various features of such black holes with nontrivial boundaries like uniqueness and stability in EGB gravity were studied by several authors [23–26]. The Birkhoff's theorem in six-dimensional EGB gravity for the case of nonconstant-curvature horizons with various features has been investigated in [27]. Also the Birkhoff's theorem is extended in Lovelock gravity for arbitrary base manifolds using an elementary method [28]. In [29] it is shown that appearing higher-curvature terms in third order Lovelock gravity, causes novel changes in the properties of the spacetime with nonconstant curvature manifold. Some exact solutions with these kinds of manifolds in Lovelock theory are presented in [30, 31]. In this article we consider Einstein manifolds of nonconstant curvature and will investigate charged solution and its properties. In Lovelock theory with $U(1)$ field, a charged black hole solution is known [32]. Also classes of nonlinear electrodynamics in Einstein and higher derivative gravity have been studied in [33–35]. For charged black holes with maximally symmetric horizons like spherical or topological black holes, stability analysis have been performed [36–39]. In [29], it is shown that uncharged black holes with nonconstant-curvature horizons have unstable phases. Our purpose is examining the response of instability of such black holes to the charge.

The paper will proceed as follows. In the next section we review the basic elements of Lovelock gravity and obtain the solution for Lovelock-Maxwell- system with nonconstant curvature horizon making use of the expressions in warped geometry for our spacetime ansatz. Also the asymptotic behaviors of the solution will be discussed. In Sec. III the expressions for the mass, temperature, entropy and electric potential of the solution are calculated. The stability analysis is also presented by calculating the free energy and heat capacity in small and large black hole limits for Ricci flat black holes in which we predict to encounter new features. Finally, we give some concluding remarks.

II. CHARGED SOLUTION WITH NONCONSTANT-CURVATURE HORIZON

We begin with the action of third order Lovelock gravity in the presence of electromagnetic field, which is written as

$$I = \int_{\mathcal{M}} d^n x \sqrt{-g} (2\Lambda + \mathcal{L}^{(1)} + \alpha_2 \mathcal{L}^{(2)} + \alpha_3 \mathcal{L}^{(3)} - F_{\mu\nu} F^{\mu\nu}). \quad (1)$$

where Λ is the cosmological constant and α_2 and α_3 are second and third order Lovelock coefficients and the Maxwell field strength, or the Faraday tensor, is given by $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the vector potential. The Einstein term $\mathcal{L}^{(1)}$ equals to R and the second order Lovelock term is $\mathcal{L}^{(2)} = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2$. Also $\mathcal{L}^{(3)}$ is the third order Lovelock Lagrangian which is described as

$$\begin{aligned} \mathcal{L}^{(3)} = & 2R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\rho\tau} R^{\rho\tau}_{\mu\nu} + 8R^{\mu\nu}_{\sigma\rho} R^{\sigma\kappa}_{\nu\tau} R^{\rho\tau}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\nu\rho} R^\rho_\mu \\ & + 3RR^{\mu\nu\sigma\kappa} R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa} R_{\sigma\mu} R_{\kappa\nu} + 16R^{\mu\nu} R_{\nu\sigma} R^\sigma_\mu - 12RR^{\mu\nu} R_{\mu\nu} + R^3. \end{aligned} \quad (2)$$

The third Lovelock term in eq. (1) has no contribution to the field equations in six or less dimensional spacetimes, we therefore consider n-dimensional spacetimes with $n > 6$. The gravitational equations following from the variation of the action (1) with respect to $g_{\mu\nu}$ reads

$$\mathcal{G}_{\mu\nu} := -\Lambda g_{\mu\nu} + G_{\mu\nu}^{(1)} + \sum_{p=2}^3 \alpha_i \left(H_{\mu\nu}^{(p)} - \frac{1}{2} g_{\mu\nu} \mathcal{L}^{(p)} \right) = \kappa_n^2 T_{\mu\nu}, \quad (3)$$

where

$$H_{\mu\nu}^{(2)} := 2(R_{\mu\sigma\kappa\tau} R^\sigma_{\nu\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} - 2R_{\mu\sigma} R^\sigma_\nu + RR_{\mu\nu}), \quad (4)$$

$$\begin{aligned} H_{\mu\nu}^{(3)} : = & -3(4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^\lambda_{\nu\tau\mu} - 8R^{\tau\rho}_{\lambda\sigma} R^{\sigma\kappa}_{\tau\mu} R^\lambda_{\nu\rho\kappa} + 2R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^{\lambda\rho}_{\tau\mu} \\ & - R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\rho} R_{\nu\mu} + 8R^\tau_{\nu\sigma\rho} R^{\sigma\kappa}_{\tau\mu} R^\rho_\kappa + 8R^\sigma_{\nu\tau\kappa} R^{\tau\rho}_{\sigma\mu} R^\kappa_\rho \\ & + 4R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\mu\rho} R^\rho_\tau - 4R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\tau\rho} R^\rho_\mu + 4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\mu} R_{\nu\rho} + 2RR_\nu^{\kappa\tau\rho} R_{\tau\rho\kappa\mu} \\ & + 8R^\tau_{\nu\mu\rho} R^\rho_\sigma R^\sigma_\tau - 8R^\sigma_{\nu\tau\rho} R^\tau_\sigma R^\rho_\mu - 8R^{\tau\rho}_{\sigma\mu} R^\sigma_\tau R_{\nu\rho} \\ & - 4RR^\tau_{\nu\mu\rho} R^\rho_\tau + 4R^{\tau\rho} R_{\rho\tau} R_{\nu\mu} - 8R^\tau_{\nu} R_{\tau\rho} R^\rho_\mu + 4RR_{\nu\rho} R^\rho_\mu - R^2 R_{\mu\nu}), \end{aligned} \quad (5)$$

and the energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = 2F^\rho_\mu F_{\rho\nu} - \frac{1}{2} F_{\rho\sigma} F^{\rho\sigma} g_{\mu\nu}. \quad (6)$$

Furthermore, variation of the action (1) with respect to the electromagnetic field reads

$$\nabla_\nu F^{\mu\nu} = 0 \quad (7)$$

Let us consider the following metric

$$g_{\mu\nu}dx^\mu dx^\nu = g_{ab}(y)dy^a dy^b + r^2(y)\gamma_{ij}(z)dz^i dz^j, \quad (8)$$

to be a warped product of a 2-dimensional *Riemannian* submanifold \mathcal{M}^2 and an $(n-2)$ -dimensional submanifold $\mathcal{K}^{(n-2)}$. In (8) $a, b = 0, 1$ and i, j go from $2, \dots, n-1$. For a spherically symmetric spacetime, the metric of \mathcal{M}^2 is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2. \quad (9)$$

We assume the submanifold $\mathcal{K}^{(n-2)}$ with the unit metric γ_{ij} to be an Einstein manifold with nonconstant curvature but having a constant Ricci scalar being

$$\tilde{R} = \kappa(n-2)(n-3), \quad (10)$$

with κ being the sectional curvature. Hereafter we use tilde for the tensor components of the submanifold $\mathcal{K}^{(n-2)}$. The Ricci and Riemann tensors of the Einstein manifold are

$$\tilde{R}_{ij} = \kappa(n-3)\gamma_{ij}, \quad (11)$$

$$\tilde{R}_{ij}{}^{kl} = \tilde{C}_{ij}{}^{kl} + \kappa(\delta_i{}^k \delta_j{}^l - \delta_i{}^l \delta_j{}^k), \quad (12)$$

where $\tilde{C}_{ij}{}^{kl}$ is the Weyl tensor of $\mathcal{K}^{(n-2)}$.

For the metric (8) to be a solution of field equations in third order Lovelock theory in vacuum, it would suffice that the Weyl tensor of the horizon satisfies the following constraints

$$\sum_{kln} \tilde{C}_{ki}{}^{nl} \tilde{C}_{nl}{}^{kj} = \frac{1}{n} \delta_i{}^j \sum_{kmnpq} \tilde{C}_{km}{}^{pq} \tilde{C}_{pq}{}^{km} \equiv \eta_2 \delta_i{}^j, \quad (13)$$

$$\begin{aligned} & \sum_{klmnp} 2(4\tilde{C}^{nm}{}_{pk} \tilde{C}^{kl}{}_{ni} \tilde{C}^{pj}{}_{ml} - \tilde{C}^{pm}{}_{ni} \tilde{C}^{jnlk} \tilde{C}_{klpm}) \\ &= \frac{2}{n} \delta_i{}^j \sum_{klmnpqr} \left(4\tilde{C}^{qm}{}_{pk} \tilde{C}^{kl}{}_{qr} \tilde{C}^{pr}{}_{ml} - \tilde{C}^{pm}{}_{qr} \tilde{C}^{rqkl} \tilde{C}_{klpm} \right) \\ &\equiv \eta_3 \delta_i{}^j. \end{aligned} \quad (14)$$

The first constraint was originally introduced by Dotti and Gleiser in [22] and the second one which is dictated by the third order Lovelock term, is obtained in [29].

In some sense η_2 and η_3 are thought to be topological charges. We are looking for the charged solutions with nonconstant curvature horizon, thus we consider the vector potential of the form

$$A_\mu dx^\mu = A_a(y) dy^a = \frac{q}{(n-3)r^{n-3}} dt \quad (15)$$

where q is an arbitrary real constant which is related to the charge of the solution. With this assumption, equation (7) is trivially satisfied.

Making use of the expressions in warped geometry, the tt component of field equation (3) is calculated to be

$$\begin{aligned} & \frac{(n-2)}{2r^6} \{ [r^5 + 2\hat{\alpha}_2 r^3 (\kappa - f) + 3\hat{\alpha}_3 r (\hat{\eta}_2 + (\kappa - f)^2)] f' - (\kappa - f) [(n-3)r^4 \\ & + (n-5)\hat{\alpha}_2 r^2 (\kappa - f) + (n-7)\hat{\alpha}_3 (3\hat{\eta}_2 + (\kappa - f)^2)] \\ & - \left((n-1)\hat{\alpha}_0 + \frac{(n-5)\hat{\alpha}_2 \hat{\eta}_2}{r^4} + \frac{(n-7)\hat{\alpha}_3 \hat{\eta}_3}{r^6} \right) r^6 \} = \mathcal{G}_t^t = -q^2 r^{10-2n} \end{aligned} \quad (16)$$

We define $\hat{\alpha}_0 = -2\Lambda/(n-1)(n-2)$, $\hat{\alpha}_2 = \frac{(n-3)!\alpha_2}{(n-5)!}$, $\hat{\alpha}_3 = \frac{(n-3)!\alpha_3}{(n-7)!}$, $\hat{\eta}_2 = \frac{(n-6)!\eta_2}{(n-2)!}$ and $\hat{\eta}_3 = \frac{(n-8)!\eta_3}{(n-2)!}$ for simplicity. We consider α_2 and α_3 as positive parameters. It is also notable to mention that $\hat{\eta}_2$ is always positive, but $\hat{\eta}_3$ can be positive or negative relating to the metric of the spacetime. For example cross product of p ($p \geq 3$) 2-spheres are Einstein spaces satisfying conditions (13) and (14) having positive $\hat{\eta}_3$ and that of 2-hyperbolas having negative $\hat{\eta}_3$. In general if K^p denotes a p -dimensional maximally symmetric space, the q th products of such spaces also satisfy conditions (13) and (14). Other non-trivial examples are the complex projective spaces like the standard Fubini-Study metric or the Bergman space which are considered in [27] in six dimensions. The interesting point of such black holes is that they can lead via Kaluza-Klein compactification to lower-dimensional scalar-tensor black holes [40, 41]. See also Ref. [30] for more examples of Einstein spaces. Introducing

$$\psi(r) = \frac{\kappa - f(r)}{r^2}, \quad (17)$$

and integrating $\int r^{n-2} \mathcal{G}_t^t dr$, one obtains

$$\left(1 + \frac{3\hat{\alpha}_3\hat{\eta}_2}{r^4}\right)\psi + \hat{\alpha}_2\psi^2 + \hat{\alpha}_3\psi^3 + \hat{\alpha}_0 + \frac{\hat{\alpha}_2\hat{\eta}_2}{r^4} + \frac{\hat{\alpha}_3\hat{\eta}_3}{r^6} - \frac{m}{r^{n-1}} + \frac{2q^2}{(n-2)(n-3)r^{2(n-2)}} = 0, \quad (18)$$

where m is the integration constant.

This cubic equation can admit three real roots. One of the real solutions to this equation may be written as:

$$\begin{aligned} \psi(r) &= -\frac{\alpha_2 r^2}{3\hat{\alpha}_3} \left\{ 1 - \left(j(r) \pm \sqrt{\gamma + j^2(r)} \right)^{1/3} + \gamma^{1/3} \left(j(r) \pm \sqrt{\gamma + j^2(r)} \right)^{-1/3} \right\}, \\ j(r) &= -1 + \frac{9\hat{\alpha}_3}{2\hat{\alpha}_2^2} - \frac{27\hat{\alpha}_3^2}{2\hat{\alpha}_2^3} \left(\hat{\alpha}_0 - \frac{m}{r^{n-1}} + \frac{\hat{\alpha}_3\hat{\eta}_3}{r^6} + \frac{q^2}{(n-3)r^{2(n-2)}} \right), \\ \gamma &= \left(-1 + \frac{3\hat{\alpha}_3}{\hat{\alpha}_2^2} + \frac{9\hat{\alpha}_3^2\hat{\eta}_2}{\hat{\alpha}_2^2 r^4} \right)^3, \end{aligned} \quad (19)$$

One may note that since the constant $\hat{\eta}_2$ and $\hat{\eta}_3$ are evaluating on the $(n-2)$ -dimensional boundary, $\hat{\eta}_3$ appears in the above equation only for $n \geq 8$. Thus in order to have the effects of non-constancy of the curvature of the horizon in third order Lovelock gravity, n should be larger than seven. One may note that solution (19) reduces to the algebraic equation of Lovelock gravity for charged solution with constant curvature horizon when $\hat{\eta}_2 = \hat{\eta}_3 = 0$.

Here we pause to add some comments on the asymptotic behavior of the solution. The behavior of the metric function f around $r \rightarrow \infty$ is exactly the same as the uncharged solution, because the term including charge vanishes at infinity. Using Eq. (17) and taking the $r \rightarrow \infty$ limit of Eq. (18), one obtains

$$(k - f_\infty)r^4 + \hat{\alpha}_2[(k - f_\infty)^2 + \hat{\eta}_2]r^2 + \hat{\alpha}_3[(k - f_\infty)^3 + 3\hat{\eta}_2(k - f_\infty) + \hat{\eta}_3] + \hat{\alpha}_0 = 0, \quad (20)$$

where the constant f_∞ is the value of f at infinity. The asymptotic AdS solution exists if Eq. (20) has positive real roots. One may note that in the case $\hat{\eta}_2 = \hat{\eta}_3 = 0$ and $k = 1$, one of the roots of Eq. (20) for $\hat{\alpha}_0 = 0$ will be $f_\infty = 1$. That is, the third order Lovelock gravity with spherical horizon can be asymptotically flat [4]. See [42, 43] for more details on the asymptotic behavior.

The Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverges at $r = 0$. Hence, there is an essential singularity located at $r = 0$. The dominant term of the metric function around $r = 0$ regarding Eq. (18) is

$$f(r) \simeq \left(\frac{2q^2}{(n-2)(n-3)\hat{\alpha}_3 r^{2n-10}} \right)^{1/3} \quad (21)$$

As one can see from the above equation the dominant term at $r = 0$ is the charge term and therefore the central singularity is always timelike, in contrast with the uncharged solution which possesses a spacelike central singularity.

As it is known, Killing horizon for a black hole is defined by $f(r_h) = 0$. In order to investigate the existence of the horizon of black hole we consider Eq. (18) for the probable existing r_h . From the definition of ψ in Eq. (17), for $\kappa \neq 0$ we substitute $r_h = (\kappa/\psi_h)^{1/2}$ to obtain

$$\begin{aligned} & m\left(\frac{\psi_h}{\kappa}\right)^{\frac{n-1}{2}} - \frac{2q^2}{(n-2)(n-3)}\left(\frac{\psi_h}{\kappa}\right)^{n-2} \\ &= \hat{\alpha}_0 + \psi_h + \hat{\alpha}_2(\kappa^2 + \hat{\eta}_2)\left(\frac{\psi_h}{\kappa}\right)^2 + \hat{\alpha}_3(\kappa^3 + 3\hat{\eta}_2\kappa + \hat{\eta}_3)\left(\frac{\psi_h}{\kappa}\right)^3 \equiv A(\psi_h) \end{aligned} \quad (22)$$

Solving this equation for m we get

$$m = \frac{2q^2}{(n-2)(n-3)}\left(\frac{\psi_h}{\kappa}\right)^{\frac{n-3}{2}} + A(\psi_h)\left(\frac{\psi_h}{\kappa}\right)^{-\frac{n-1}{2}} \equiv B(\psi_h) \quad (23)$$

The solutions of this equation give the horizon radius. Solving $\partial_\psi B(\psi) = 0$, we obtain

$$q^2 = \frac{(n-2)}{2} \left\{ \hat{\alpha}_0(n-1)\left(\frac{\psi}{\kappa}\right)^{2-n} + \kappa(n-3)\left(\frac{\psi}{\kappa}\right)^{3-n} + (n-5)\hat{\alpha}_2(\kappa^2 + \hat{\eta}_2)\left(\frac{\psi}{\kappa}\right)^{4-n} \right. \quad (24)$$

$$\left. + (n-7)\hat{\alpha}_3(\kappa^3 + 3\hat{\eta}_2\kappa + \hat{\eta}_3)\left(\frac{\psi}{\kappa}\right)^{5-n} \right. \quad (25)$$

$$\equiv C(\psi) \quad (26)$$

We plot function $C(\psi)$ versus ψ . The cross point of the curve $C(\psi)$ with q^2 is the real root for the equation $C(\psi) = q^2$ that is shown in Fig. (1). We call it ψ_{\min} for which the function $B(\psi)$ has its extreme value. Figure (2) shows $B(\psi)$ versus ψ with its minimum at $\psi = \psi_{\min}$. The intersection of the horizontal line m and this curve gives the radius of the horizon. There exist horizons if $m \geq m_{ext}$ where m_{ext} is defined as

$$m_{ext} = \frac{q^2}{(n-3)}\psi_{\min}^{\frac{n-3}{2}} + A(\psi_{\min})\psi_{\min}^{-\frac{n-1}{2}}. \quad (27)$$

It is worth noting that the Eq. (23) may have real roots if we set $m = 0$. This fact is due to the existence of $\hat{\eta}_3$ that could be negative in the equations (22) and (23). Thus charged black holes with $m = 0$ and nonconstant-curvature horizon may have horizon. This does not happen for the solution with constant-curvature horizon or the solution with nonconstant-curvature horizon in second order Lovelock theory. Also, the reader notes that m_{ext} could be negative for negative $\hat{\eta}_3$, and therefore $m = 0$ is larger than m_{ext} which is negative.

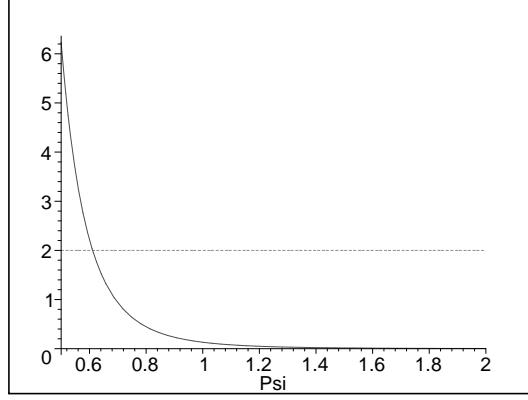


FIG. 1: $C(\psi)$ (line) and q^2 (dotted) versus ψ for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = 0.006$, $q = 2$.

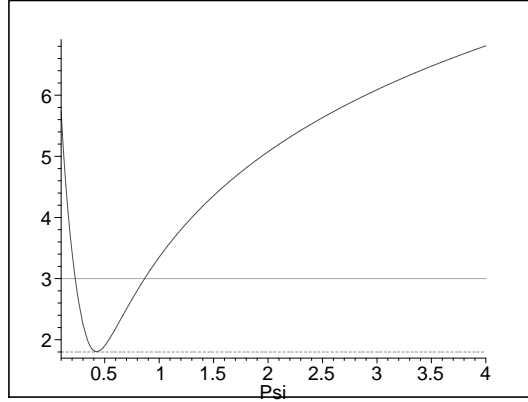


FIG. 2: $B(\psi)$ (line) and m_{ext} (dotted) versus ψ for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = 0.006$, $q = 2$. It can be seen that there exist horizons if $m \geq m_{ext}$.

III. THERMODYNAMICS OF BLACK HOLE SOLUTIONS

The surface gravity on the Killing horizon is $(1/2)(df/dr)|_{r=r_h}$, from which the temperature of the horizon T could be written as

$$T = \frac{(n-1)r_h^6\hat{\alpha}_0 + (n-3)\kappa r_h^4 + (n-5)\hat{\alpha}_2(\hat{\eta}_2 + \kappa^2)r_h^2 + (n-7)\hat{\alpha}_3(\hat{\eta}_3 + 3\kappa\hat{\eta}_2 + \kappa^3) - \frac{2q^2}{(n-2)}r^{10-2n}}{4\pi r_h[r_h^4 + 2\kappa\hat{\alpha}_2r_h^2 + 3\hat{\alpha}_3(\hat{\eta}_2 + \kappa^2)]}, \quad (28)$$

where r_h is the radius of the outer horizon. On the other hand, the entropy on the Killing horizon is calculated using the Wald prescription which is applicable for any black hole solution of which the event horizon is a killing horizon [44]. The Wald entropy is defined by

the following integral performed on $(n - 2)$ -dimensional spacelike bifurcation surface

$$S = -2\pi \oint d^{n-2}x \sqrt{h} Y, \quad Y = Y^{abcd} \hat{\varepsilon}_{ab} \hat{\varepsilon}_{cd}, \quad Y^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}} \quad (29)$$

in which \mathcal{L} is the Lagrangian and $\hat{\varepsilon}_{ab}$ is the binormal to the horizon. As we mentioned before, \mathcal{L}_1 , \mathcal{L}_2 and \mathcal{L}_3 , are Einstein, Gauss-Bonnet and third order Lovelock Lagrangians respectively, from which we obtain Y_1 , Y_2 and Y_3 . Following the given description, Y_1 and Y_2 and Y_3 are calculated to be

$$Y_1 = -\frac{1}{8\pi} \quad (30)$$

$$Y_2 = -\frac{\hat{\alpha}_2}{4\pi} [R - 2(R_t^t + R_r^r) + 2R_{tr}^{tr}] \quad (31)$$

$$\begin{aligned} Y_3 = & -\frac{3\hat{\alpha}_3}{4\pi} \{ -12(R_{tm}^{tm} R_{rn}^{rn} - R_{rn}^{tm} R_{mt}^n) + 12R^{trmn} R_{trmn} - 24[R_{tm}^{tr} R_r^m - R_{rm}^{tr} R_t^m] \\ & + \frac{1}{4}(R_{mnpr} R^{mnp r} + R_{mnp t} R^{mnp t}) + 3(2R R_{tr}^{tr} + \frac{1}{2} R_{mnpq} R^{mnpq}) \\ & + 12(R_t^t R_r^r - R_r^t R_t^r + R_{mrn}^r R^{mn} + R_{mtn}^t R^{mn}) + 12(R^{rm} R_{rm} + R^{tm} R_{tm}) \\ & - 6[R_{mn} R^{mn} + R(R_r^r + R_t^t)] + \frac{3}{2} R^2 \}. \end{aligned} \quad (32)$$

Substituting in Eq. (29), and making use of Eq. (12), one calculates the entropy to be

$$S = -2\pi \{Y_1 + Y_2 + Y_3\} = \frac{r_h^{n-2}}{4} \left\{ 1 + \frac{2\kappa \hat{\alpha}_2 (n-2)}{r_h^2 (n-4)} + \frac{3\hat{\alpha}_3 (n-2)(\hat{\eta}_2 + \kappa^2)}{r_h^4 (n-6)} \right\}. \quad (33)$$

The charge of the black holes per unit volume can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{q}{4\pi}. \quad (34)$$

The electric potential, measured at infinity with respect to the horizon, is defined by [45]

$$\Phi = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r \rightarrow r_h}. \quad (35)$$

Using $\chi = \partial/\partial t$ as the null generator of the horizon, one finds

$$\Phi = \frac{q}{(n-3)r_h^{n-3}}. \quad (36)$$

Also we obtain the relation for the mass density, from Eqs. (18) and (17), which admits the relation below

$$M = \frac{(n-2)m}{16\pi} = \frac{(n-2)}{16\pi} [\hat{\alpha}_0 r^{n-1} + \kappa r^{n-3} + \hat{\alpha}_2 [\kappa^2 + \hat{\eta}_2] r^{n-5} + \hat{\alpha}_3 [\kappa^3 + 3\hat{\eta}_2 \kappa + \hat{\eta}_3] r^{n-7} + \frac{2q^2}{(n-2)(n-3)r^{n-3}}]. \quad (37)$$

Making use of Eqs. (28), (33), (34) and (37), one may note that the thermodynamic quantities calculated in this section satisfy the first law of thermodynamics $dM = T\partial S + \Phi\partial Q$.

IV. STABILITY IN CANONICAL ENSEMBLE FOR THE CASE OF $\kappa = 0$

The stability analysis of a thermodynamic system with respect to the small variations of the thermodynamic coordinates, is performed by analyzing the behavior of the entropy near equilibrium. The number of thermodynamic variables depends on the ensemble that is used. In the canonical ensemble, the charge is fixed, and therefore positive heat capacity, $C = T(\partial S/\partial T)_Q$, implies that the black hole is locally stable. However, to analyze the global stability, we should check the free energy of the black hole which is defined by $F := M - TS$, whereby negative value ensures global stability [46]. Before investigating the behavior of heat capacity, we check the behavior of temperature for small and large black holes. As mentioned before, it is seen from Eq. (28), that extreme black hole exists when $T(r_h) = 0$, from which we get:

$$q_{ext} = r^{n-5} \sqrt{\frac{(n-2)}{2} [(n-1)r_h^6 \hat{\alpha}_0 + (n-5)\hat{\alpha}_2 \hat{\eta}_2 r_h^2 + (n-7)\hat{\alpha}_3 \hat{\eta}_3]}. \quad (38)$$

As we mentioned before, $\hat{\eta}_3$ can be positive or negative relating to the metric of the spacetime. Therefore we consider these two cases separately:

a) The case $\hat{\eta}_3 > 0$:

In this case q_{ext} is always real and extreme black hole exists for $q = q_{ext}$. To investigate the stability of the black holes in this case we check the positivity of $(\partial T/\partial S)_Q$ in the regions where T is positive, because the temperature of a physical black hole is positive. We plot the curve of T and $(\partial T/\partial S)_Q$ versus the radius for very small black holes in Fig. (3) and for

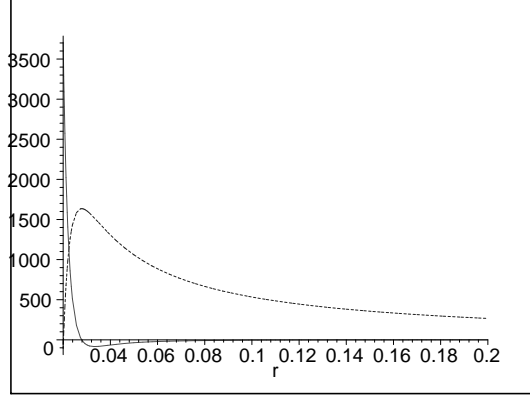


FIG. 3: $10^{-3}(\partial T/\partial S)_Q$ (line) and $10^2 T$ (dotted) versus r_h for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = 0.006$, $q = 1$.

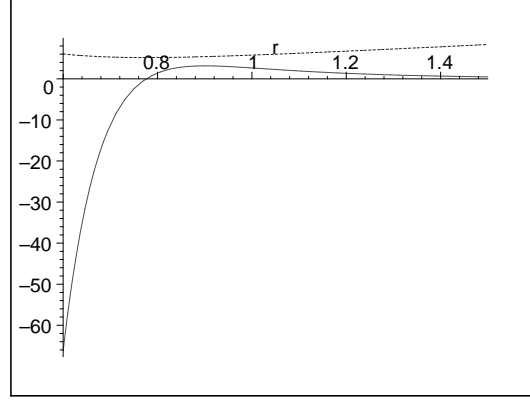


FIG. 4: $(\partial T/\partial S)_Q$ (line) and T (dotted) versus r_h for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = 0.006$, $q = 1$.

small and large black holes in figure (4). It is seen that there is a phase transition between very small and small black holes from a stable to an unstable phase, which is characterized by the sign change in heat capacity. This is due to the existence of charge in the solution and does not occur in uncharged solution with nonconstant curvature horizon [29]. Large black holes have positive heat capacity and are locally stable. To perform the analysis of global stability we depict F versus r in figure (5), from which we note that small black holes are globally unstable while large ones are stable.

b) The case $\hat{\eta}_3 < 0$:

For this case $\hat{\eta}_3$ should satisfy the following condition

$$|\hat{\eta}_3| < \frac{(n-1)r_h^6\hat{\alpha}_0 + (n-5)\hat{\alpha}_2\hat{\eta}_2r_h^2}{(n-7)\hat{\alpha}_3}. \quad (39)$$

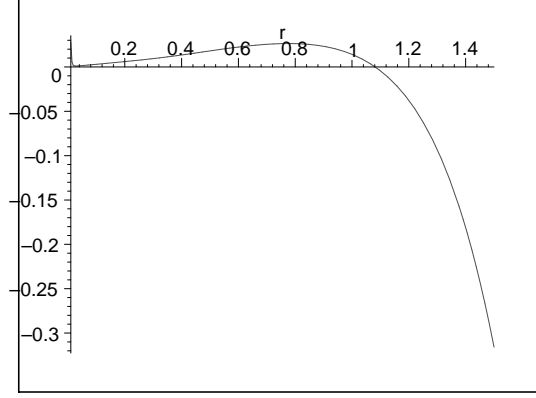


FIG. 5: F versus r_h for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = 0.006$, $q = 1$.

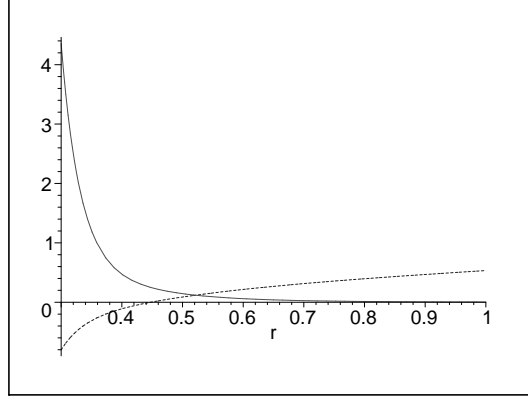


FIG. 6: $(\partial T/\partial S)_Q$ (line) and T (dotted) versus r_h for $n = 8$, $\hat{\alpha}_0 = 1$, $\hat{\alpha}_2 = 0.2$, $\hat{\alpha}_3 = 0.05$, $\hat{\eta}_2 = 0.5$ and $\hat{\eta}_3 = -2$, $q = 1$.

in order to have a real value for q_{ext} . We plot the curve of T and $(\partial T/\partial S)_Q$ versus r in figure (6) for $q < q_{ext}$ as temperature is positive in this case. It is seen that $(\partial T/\partial S)_Q$ is positive in the range that T is positive and thereby, charged black holes with negative $\hat{\eta}_3$, are locally stable. The plot of free energy and analysis of global stability in this case are like the previous case.

V. CONCLUDING REMARKS

In this study, we presented charged solution of Lovelock gravity with nonconstant-curvature horizon. We considered a spacetime which is a cross product of a Lorentzian spacetime and a space with nonconstant-curvature horizon. With this assumption the usual assumption of maximally symmetry is relaxed, leading to a new class of black hole solution

which introduces novel chargelike parameter to the black hole potential, in addition to the electric charge. These parameters are obtained by imposing two conditions on the Weyl tensor of the Einstein space and appear with the advantage of higher curvature terms in third order Lovelock equations. It was shown that the electric charge dominates the behavior of the metric function as r approaching zero, and the central singularity is always timelike, in contrast with the uncharged solution which possesses a spacelike central singularity. By investigating the asymptotic behavior of the metric at infinity, we showed that the solution could be asymptotically AdS in contrast with the solutions with constant-curvature horizon that could be flat for $\widehat{\alpha}_0 = 0$ and $\kappa = 1$. By introducing the condition for the existence of the event horizon of the solution, we mentioned that charged black holes with nonconstant-curvature horizon and $m = 0$ could possess event horizon. This property does not appear for black hole with constant-curvature horizon or the one with nonconstant-curvature horizon in second order Lovelock theory. Then we proceeded by calculating the mass, temperature, entropy and electric potential of the black hole in terms of horizon radius and the first law of thermodynamics was shown to hold for this class of solution. In order to show the differences of charged and uncharged black holes with nonconstant-curvature horizons, we went through the properties of black holes with $\kappa = 0$. We saw that extreme charged black hole could exist depending on the values of electric charge q and $\widehat{\eta}_3$ appearing in the expression for temperature. Also we mentioned that the entropy of the black hole with $\kappa = 0$ does not obey the area law exactly like that of uncharged solution. To check the stability of the solutions, confining to the canonical ensemble, we calculated the heat capacity and free energy in the small and large black hole regimes. The main difference that occurs due to the existence of charge is that $\kappa = 0$ charged solution with nonconstant-curvature horizon for $\widehat{\eta}_3 > 0$ shows a transition between a thermodynamically stable phase for very small black holes to a thermodynamically unstable phase for small ones which does not appear for uncharged black holes. Also calculations show that small charged black holes have positive free energy and are globally unstable while large black holes possess negative free energy and positive heat capacity showing their stability to both perturbative and non-perturbative fluctuations.

Acknowledgments

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